Weak Phase γ and Strong Phase δ from
CP Averaged $B \to \pi\pi$ and πK Decays

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Abstract

Assuming $SU(3)$ symmetry for the strong phases in the four decay modes $B \to \pi^-\pi^+, \pi^0\pi^+, \pi^-K^+, \pi^-\bar{K}^0$ and ignoring the relative small electroweak penguin effects in those decays, the weak phase γ and the strong phase δ can be determined in a model independent way by the CP-averaged branching ratios of the four decay modes. It appears that the current experimental data for $B \to \pi\pi$ and πK decays prefer a negative value of $\cos\gamma\cos\delta$. By combining with the other constraints from V_{ub}, $B^0_{d,s} - \bar{B}^0_{d,s}$ mixings and indirect CP-violating parameter ϵ_K within the standard model, two favorable solutions for the phases γ and δ are found to lie in the region: $35° \lesssim \gamma \lesssim 62°$ and $106° \lesssim \delta \lesssim 180°$ or $86° \lesssim \gamma \lesssim 151°$ and $0° \lesssim \delta \lesssim 75°$ within 1σ standard deviation. It is noted that if allowing the standard deviation of the data to be more than 1σ, the two solutions could approach to one solution with a much larger region for the phases γ and δ. Direct CP asymmetry $a^{(\pi^-K^+)}_{\epsilon''}$ in $B \to \pi^-K^+$ decay can be as large as the present experimental upper bound. Direct CP asymmetry $a^{(\pi^+\pi^-)}_{\epsilon''}$ in $B \to \pi^-\pi^+$ decay can reach up to about 40% at 1σ level.
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The study of CP-violation is one of the central topics in the present day elementary particle physics. In the standard model (SM), all the CP violating phenomena arise from a single complex phase of the Cabbibo-Kobayashi-Maskawa(CKM) matrix elements. If the CKM phase is the only source of CP violation, some unitarity relations such as $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ will hold . The unitarity relations can be represented geometrically by a set of triangles called unitarity triangles . The three angles in the triangle containing b and d quarks are often denoted by α, β and γ with $\alpha + \beta + \gamma = \pi$ in the SM. Thus one of the important issues is to precisely determine those angles and their sum. Any deviation of the sum from π will be a signal of new physics [1].

Although angles α and β may be well measured via the time dependent measurements of $B \to \psi K_S$ and $B \to \pi\pi$, the determination of angle γ is a great challenge for both theorists and experimentists. In the recent years much work has been done on this issue [2–5]. As it was first proposed in Ref. [2], the angle γ may be extracted through six $B \to DK$ decay rates. The difficulty of this method is that it needs tagging of the CP eigenstate D_{CP}^0 which is rather difficult in the experiment. It may also be extracted from the decay mode $B_s^0 \to (D_s^- K^+, D_s^+ K^-) \leftarrow \bar{B}_s^0$ in a model-independent way [3] since one only needs to extract the rephase invariant observables $a_{\epsilon+\epsilon'}$ and $a_{\epsilon'}$ from a time-dependent measurement. Thus the weak phase is simply given by $\sin\gamma = a_{\epsilon+\epsilon'}/\sqrt{1+a_{\epsilon'}^2}$. In the recent years an alternative way of using the CP averaged $B \to \pi^{\pm}K^0, \pi^+\pi^0$ and the CP conjugate $B^+ \to \pi^0 K^+$, $B^- \to \pi^0 K^-$ branching ratios has been aroused a great attention [4]. However this method needs some theoretical input in evaluating the electroweak penguin (EWP) effects. At present, limited by the statistics the difference of CP conjugate rates can not be definitely established [6]

Recently, the CLEO Collaboration has reported the first observation of rare decays $B \to \pi^-\pi^+$ and $\pi^0\bar{K}^0$ [7]. The observation of $\pi^0\bar{K}^0$ complete the set of measurements on $B \to \pi K$ decays. Other channels of πK have also been largely improved. The most recent results reported by CLEO collab. are (in units of 10^{-6}) [8],

$$Br(B \to \pi^-\pi^+) = 4.3^{+1.6}_{-1.4} \pm 0.5$$

$$Br(B \to \pi^0\pi^+) = < 12.7 \quad (5.6^{+2.6}_{-2.3} \pm 1.7)$$

$$Br(B \to \pi^-K^+) = 17.2^{+2.5}_{-2.4} \pm 1.2$$

$$Br(B \to \pi^-\bar{K}^0) = 18.2^{+4.6}_{-4.0} \pm 1.6$$

$$Br(B \to \pi^0 K^+) = 11.6^{+3.0+1.4}_{-2.7-1.3}$$

$$Br(B \to \pi^0\bar{K}^0) = 14.6^{+5.9+2.4}_{-5.1-3.3} \tag{1}$$

Although only the upper bound of $\pi^+\pi^0$ is given, the CLEO Collab. also quote a value of $Br(B \to \pi^0\pi^+) = 5.6^{+2.6}_{-2.3}\pm 1.7$. This will be improved by the future measurements. The relative small value of $\pi^-\pi^+$, the almost equal $K\pi$ rates: $\pi^-\bar{K}^0 \approx \pi^-K^+$ and large $\pi^0\bar{K}^0$ seem to be in conflict with the theoretical predictions. However, as it was pointed out in Ref. [9], if one takes the weak phase γ of the CKM matrix elements to be larger than 90° and include the EWP effects, the situation for π^0K^+ may be improved greatly, but for $\pi^0\bar{K}^0$ it may become worse as the EWP-SP (strong penguin) interference in $\bar{K}^0\pi^0$ decay is likely to be destructive. Some alternative ways in solving this puzzle are also proposed, such as the small $|V_{ub}|$ in $B \to \pi^+\pi^-$ [10] and the use of different form factors [11] and the possibility of large final state interaction phase [12]. It may also be interesting to consider the new physics effects in those decay modes [13].

Note that the theoretical description on nonleptonic B decays is model dependent. Although the short-distance effects are calculable from the Wilson coefficients, one has to assume factorization approach and adopt some models in evaluating the long-distance effects. It may then concern many phenomenological parameters, such as the decay constant of B meson, the transition form factors as well as the effective color number N_C, which still suffer from large uncertainties. Thus the precision of theoretical calculations is unfortunately limited.

In this paper, we shall consider some less model dependent ways to extract both the weak phase γ and the strong phase δ due to final state interactions. The basic point is to assume approximate relations among the strong phases and choose the decay modes with relative small

EWP effects so that one could ignore their contributions as the first step approximation. For this purpose, we take the following four interesting decay modes: $B \to \pi^-\pi^+, \pi^0\pi^+, \pi^-K^+, \pi^-\bar{K}^0$. It will be seen that, under the above assumptions and considerations, the four CP-averaged branching ratios could be used to extract the phases γ and δ as well as the relative contributions between strong penguin (SP) graphs and tree graphs without additional theoretical inputs. Though such a treatment still suffers from some uncertainties, it could directly provide us useful constraints and insight on the phases γ and δ. We will show that at the 1σ level of the current experimental data, there exist two correlated regions between γ and δ, which are corresponding to two solutions of negative $\cos\gamma\cos\delta$, i.e., one solution is with positive $\cos\gamma$ but negative $\cos\delta$, another with negative $\cos\gamma$ but positive $\cos\delta$. While at more than 1σ level a much larger region for the phases γ and δ is allowed.

Generally, the B decay amplitude can be decomposed by several $SU(3)$ invariant Fenyman diagrams [14,15]. In this decomposition one may see that the amplitudes of decay $B \to \pi\pi$ and $B \to \pi K$ are correlated. This can be used to study the penguin effectcs as well as the strong phases in those modes [16,17]. In $SU(3)$ limits, the decay amplitudes of $B \to \pi^-\pi^+, \pi^0\pi^+, \pi^-K^+, \pi^-\bar{K}^0$ have the following forms [18,19]

$$\mathcal{A}(B \to \pi^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{ud}V_{ub}^*(-\frac{T+C}{\sqrt{2}}) \tag{2}$$

$$\mathcal{A}(B \to \pi^-\pi^+) = \frac{G_F}{\sqrt{2}}[V_{ud}V_{ub}^*(T-P) - V_{cd}V_{cb}^*P] \tag{3}$$

$$\mathcal{A}(B \to \pi^-\bar{K}^0) = \frac{G_F}{\sqrt{2}} V_{ts}V_{tb}^*P' \tag{4}$$

$$\mathcal{A}(B \to \pi^-K^+) = \frac{G_F}{\sqrt{2}}[V_{us}V_{ub}^*T' + V_{ts}V_{tb}^*P'] \tag{5}$$

where the factor $1/\sqrt{2}$ in Eq.(2) comes from the π^0 wave function. $T, T'(C)$ and P, P' denote the Tree(Color suppressed) and QCD penguin amplitude with different strong phases:

$$T = |T|e^{i\delta_T}, \quad P = |P|e^{i\delta_P}$$

$$T' = |T'|e^{i\delta_T'}, \quad P' = |P'|e^{i\delta_P'} \tag{6}$$

In the expression for the amplitude $\mathcal{A}(B \rightarrow \pi^- \pi^+)$ in Eq.(3), the unitarity relation of CKM matrix elements $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ has been used to remove the factor $V_{td}V_{tb}^*$ which comes from the inner t-quark of the QCD penguins. This allows us to extract the weak phase γ instead of α, which is different from the usual treatments [20,21].

The charge conjugate decay amplitude can be obtained by simply inverting the sign of weak phase γ. We then get the CP- averaged branching ratios:

$$Br(B \rightarrow \pi^+ K^0) = \frac{1}{2}\left(B^0 \rightarrow \pi^+ K^0 + \bar{B}^0 \rightarrow \pi^- \bar{K}^0\right) \simeq |V_{ts}V_{tb}^*|^2 |P'|^2 \ , \tag{7}$$

$$Br(B \rightarrow \pi^+ \pi^0) = \frac{1}{2}\left(B^0 \rightarrow \pi^+ \pi^0 + \bar{B}^0 \rightarrow \pi^- \pi^0\right) \simeq \frac{1}{2}|V_{ud}V_{ub}^*|^2 |T + C|^2 \ , \tag{8}$$

$$Br(B \rightarrow \pi^- K^+) = \frac{1}{2}\left(Br(B^0 \rightarrow K^+ \pi^-) + Br(\bar{B}^0 \rightarrow K^- \pi^+)\right)$$

$$\simeq |V_{us}V_{ub}^*|^2 |T'|^2 - 2|V_{us}V_{ub}^*||V_{ts}V_{tb}^*| |T'P'| \cos\delta \cos\gamma + |V_{ts}V_{tb}^*|^2 |P'|^2 \ , \tag{9}$$

$$Br(B \rightarrow \pi^- \pi^+) = \frac{1}{2}\left(Br(B^0 \rightarrow \pi^+ \pi^-) + Br(\bar{B}^0 \rightarrow \pi^+ \pi^-)\right)$$

$$\simeq |V_{ud}V_{ub}^*|^2 |(|T| e^{i\delta} - |P|)|^2$$

$$+ 2|V_{ud}V_{ub}^*||V_{cd}V_{cb}^*| |TP| \left(\cos\delta \cos\gamma - \left|\frac{P}{T}\right| \cos\gamma\right)$$

$$+ |V_{cd}V_{cb}^*|^2 |P|^2 \ . \tag{10}$$

In writing down the above equations, we have neglected the EWP effects. The EWP effects are often thought to be very important [22] in the $B \rightarrow \pi K$ decays, but it remains depending on different decay modes. In deed, the EWP effects are of crucial importance in the decay modes: $B \rightarrow \pi^0 K^0$ and $\pi^0 \pi^0$ in which the contributions from the tree diagrams are color suppressed. While in the decay modes: $B \rightarrow \pi^- \pi^+, \pi^0 \pi^+, \pi^- K^+, \pi^- \bar{K}^0$, the EWP effects are relatively small as the contributions from tree diagrams are not color suppressed. As there remain large errors in the current experimental data, for simplicity, we may ignore the EWP effects in those four decay modes as a good approximation in comparison with the experimental uncertainties. To have a quantitative estimation of how good of the approximation, it may be seen from the model dependent calculations [23], where the contributions from the EWP graphs were found to be about $1\%, 5\%, 5\%, 8\%$ in the decay modes $B \rightarrow \pi^- \pi^+, \pi^0 \pi^+, \pi^- K^+, \pi^- \bar{K}^0$, respectively. It is

not difficult to recognize that the relative contributions of the EWP to SP graphs is about 8%, the relative contributions of the tree diagrams to the SP graphs is about 40% in the $B \to \pi^- K^+$ decay and is dominated in the $B \to \pi^- \pi^+$ decay.

It is useful to consider the ratios of the decay rates. Let us define

$$R_1 \equiv \frac{Br(B \to \pi^- \bar{K}^0)}{Br(B \to \pi^0 \pi^+)} > 1.52 \quad (3.25 \pm 1.94) \tag{11}$$

$$R_2 \equiv \frac{Br(B \to \pi^- \bar{K}^0)}{Br(B \to \pi^- K^+)} = 1.06 \pm 0.32 \tag{12}$$

$$R_3 \equiv \frac{Br(B \to \pi^- \pi^+)}{Br(B \to \pi^- K^+)} = 0.25 \pm 0.1 \tag{13}$$

In a naive estimation, the ratio between color suppressed diagram and the tree diagram, i.e. $|C/T|$ is of the order $\mathcal{O}(0.3)$ from the color suppression. However, the model dependent calculation show a very small value:$|C/T| \simeq a_2/a_1 \simeq 0.05$ when N_C is near 3 [23]. By adopting the recent analysis from Ref. [24] which is based on the heavy quark limit, we have $|C/T| \simeq 0.08$.

In a good approximation, (i.e. neglecting the terms proportional to $|V_{us} V_{ub}^* T/(V_{ts} V_{tb}^* P)|^2 \approx \mathcal{O}(10^{-2})$ in πK modes.) $\cos\gamma\cos\delta$ can be solved from the definitions of R_2 :

$$\cos\gamma\cos\delta \simeq \frac{1}{2} \left| \frac{V_{ts} V_{tb}^*}{V_{us} V_{ub}^*} \right| \left| \frac{P'}{T'} \right| \left(1 - \frac{1}{R_2} \right) \tag{14}$$

On with including the leading $SU(3)$ breaking factor f_π/f_K in the sense of generalized factorization, one then has

$$\frac{|P|}{|T|} = \frac{|P'|}{|T'|}, \qquad \frac{|T|}{|T'|} = \frac{f_\pi}{f_K} \tag{15}$$

and

$$\delta_T = \delta_{T'} \ , \ \delta_P = \delta_{P'} \tag{16}$$

Under this approximation, it is then easily seen that the ratio $|P/T|$ can be estimated from R_1 [18,25]:

$$\left| \frac{P}{T} \right| \simeq 1.09 \times \frac{f_\pi}{f_K} \sqrt{\frac{R_1}{2}} \frac{|V_{ud} V_{ub}^*|}{|V_{ts} V_{tb}^*|} > 0.055 \tag{17}$$

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when taking the central value for the mode $Br(B \to \pi^0 \pi^+) = 5.6$, we have $|P/T| = 0.08$. The value of $|P/T|$ can also be evaluated from the effective Hamitonian and be simply given only by the short distance Wilson coefficients [22,21] once adopting the factorization approach for the hadronic matrix elements

$$\frac{P}{T} = \frac{1}{a_1}\left[a_4 + a_{10} + (a_6 + a_8)\frac{2m_\pi^2}{(m_b - m_u)(m_u + m_d)}\right] \tag{18}$$

which is found to be 0.05 for $N_C=3$ and $m_u + m_d = 1.5$ MeV. Since the validity of Eq.(18) only depends on the assumption of factorization, the ratio $|P/T|$ extracted in this way is helpful to examine how goodness of the factorization approach. It seems that the current experimental data prefer a larger $|P/T|$. This needs to be further confirmed by future experiments.

To naively see the changes of the sign of $\cos\gamma\cos\delta$ as R_3 and R_1, one may neglect the terms of the order $\mathcal{O}(|P/T|^2)$ in $\pi\pi$ decay modes and use the modified $SU(3)$ relations. Then $\cos\gamma\cos\delta$ can be simply given in terms of R_1 and R_3

$$\begin{aligned}
\cos\gamma\cos\delta &\simeq 1.09 \times \left(\frac{\sqrt{2R_1}}{4}\right)\frac{R_3 - 1.68/R_1}{\frac{|V_{cd}V_{cb}^*|-|V_{ud}V_{ub}^*|}{|V_{ts}V_{tb}|} + \frac{|V_{us}V_{ub}^*|}{|V_{ud}V_{ub}^*|}\frac{f_K}{f_\pi}R_3} \\
&\simeq 1.09 \times \left(\frac{\sqrt{2R_1}}{4\lambda}\right)\frac{R_3 - 1.68/R_1}{\frac{f_K}{f_\pi}R_3 + \lambda - \left|\frac{V_{ub}}{V_{cb}}\right|}
\end{aligned} \tag{19}$$

where $\lambda = 0.22$ is the Wolfenstein parameter. This shows that $\cos\gamma\cos\delta$ will change sign when R_3 and R_1 satisfy the approximate relation $R_3 \simeq 1.68/R_1$. The precise numerical values of R_3 and R_1 for changing the sign of $\cos\gamma\cos\delta$ may be seen from Fig.1. The values of R_3 is slightly higher than the ones by Eq.(19).

With the above considerations, the phases γ and δ can be extracted from R_1, R_2 and R_3 . As the equations are quadratic in $\cos\gamma$ and $\cos\delta$, there exists a twofold ambiguity in determining these two phases. In Fig.1, we present a contour plot for R_2 and R_3 in the $\cos\gamma - \cos\delta$ plane with R_1 being fixed at 3.0, 4.5, 6.0, 7.5. Where the solid and dashed contours correspond to different values of R_2 and R_3. The points at which the two kind of curves intersect are the solutions of $\cos\gamma$ and $\cos\delta$. It can be seen from Fig.1 that these contours change largely for

different values of R_2 and R_3. When $R_2 < 1$ the contours of R_2 and R_3 are all in the II and IV quadrants. When $R_2 > 1$ the contours move into the I and III quadrants. This behavior can be understood from Eq.(14). Thus $\cos\gamma\cos\delta$ will change sign when R_2 moves across the point $R_2 = 1$. The changes of R_3 contours also have the similar reason. From the present data within 1σ standard deviation, R_2 and R_3 are in the range $0.74 \lesssim R_2 \lesssim 1.38$ and $0.15 \lesssim R_3 \lesssim 0.35$, respectively . Since R_3 is smaller than 0.35 at 1σ level, $\cos\gamma\cos\delta$ will be negative for small R_1. Namely a negative $\cos\gamma$ corresponds to a strong phase δ in the first quadrant, for positive $\cos\gamma$, the angle δ becomes large and takes values in the second quadrant. From Fig.1, one may see that for large R_1, a positive solution of $\cos\gamma\cos\delta$ is also allowed. For $R_1 \gtrsim 0.75$, the allowed range of $\cos\gamma$ and $\cos\delta$ becomes large and lies in the region: $0.2 \lesssim \cos\gamma \lesssim 1$ and $-1 \lesssim \cos\delta \lesssim 1$ or $-1 \lesssim \cos\gamma \lesssim 0.1$ and $-1 \lesssim \cos\delta \lesssim 1$.

The constraints on the phase γ may also come from other experiments, such as $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing, CP-violating parameter ϵ_K in the kaon decay, and CKM matrix element V_{ub} from semileptonic $b \to u$ decays. Combining all the constraints together and taking the branching ratio for the $B \to \pi^+\pi^0$ decay to be $Br(B \to \pi^+\pi^0) = 5.6^{+2.6}_{-2.3} \pm 1.7$, the allowed region for γ is shown in Fig.2.

It is found that the allowed range for γ is: $35° \lesssim \gamma \lesssim 62°$ or $86° \lesssim \gamma \lesssim 151°$, the corresponding values for the phase δ could range from $106°$ to $180°$ or from $0°$ to $75°$. The allowed regions for the phases γ and δ are plotted in Fig. 3 and given by the two shadowed ones. One can see from the figure that large region for $\cos\gamma$ and $\cos\delta$ has been exluded from R_1, R_2 and R_3 when they are at 1σ level.

Recently, CLEO collaboration has also reported the data on direct CP violation in $B \to \pi^- K^+$ decay. Let us now consider CP asymmetries in both $B \to \pi\pi$ and $B \to \pi K$ decays. They are defined as

$$a_{\epsilon''}^{(\pi^- K^+)} = \frac{\Gamma(\bar{B}^0 \to \pi^- K^+) - \Gamma(B^0 \to \pi^+ K^-)}{\Gamma(\bar{B}^0 \to \pi^- K^+) + \Gamma(B^0 \to \pi^+ K^-)}$$
$$= \left(2|V_{us}V_{ub}^*V_{ts}V_{tb}^*| \left|\frac{P}{T}\right| \sin\gamma \sin\delta\right)$$

$$\times \left(|V_{us}V_{ub}^*|^2 - 2|V_{us}V_{ub}^*V_{ts}V_{tb}^*| \left|\frac{P}{T}\right| \cos\gamma\cos\delta + |V_{ts}V_{tb}^*|^2 \left|\frac{P}{T}\right|^2 \right)^{-1}, \tag{20}$$

$$a_{\epsilon''}^{(\pi^+\pi^-)} = \frac{\Gamma(\bar{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^+\pi^-)}{\Gamma(\bar{B}^0 \to \pi^+\pi^-) + \Gamma(B^0 \to \pi^+\pi)}$$

$$= \left(2|V_{ud}V_{ub}^*V_{cd}V_{cb}^*| \left|\frac{P}{T}\right| \sin\gamma\sin\delta \right)$$

$$\times \left(|V_{ud}V_{ub}^*|^2 (1 - 2\left|\frac{P}{T}\right| \cos\delta + \left|\frac{P}{T}\right|^2) + |V_{cd}V_{cb}^*|^2 \left|\frac{P}{T}\right|^2 \right.$$

$$\left. +2|V_{ud}V_{ub}^*V_{cd}V_{cb}^*| \left|\frac{P}{T}\right| \cos\gamma(\cos\delta - \left|\frac{P}{T}\right|) \right)^{-1} \tag{21}$$

Here we have used the notation for the general rephase-invariant CP-violating observables classified in [3,26]. As $|P/T|$ is at order of $\mathcal{O}(10^{-1})$, for an approximate estimation, one may neglect the $|P/T|$ terms in the denominator, thus the above formulae are simplified

$$a_{\epsilon''}^{(\pi^-K^+)} \approx 2\frac{|V_{us}V_{ub}^*|}{|V_{ts}V_{tb}^*|} \left|\frac{T}{P}\right| \sin\gamma\sin\delta \tag{22}$$

$$a_{\epsilon''}^{(\pi^+\pi^-)} \approx 2\frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} \left|\frac{P}{T}\right| \sin\gamma\sin\delta$$

$$\approx 0.59 \times \frac{f_\pi^2}{f_K^2} R_1 a_{\epsilon''}^{(\pi^-K^+)} \tag{23}$$

which implies that $a_{\epsilon''}^{(\pi^+\pi^-)}$ may become large with R_1 increasing. From the data reported by the CLEO collaboration, no significant deviation from zero was observed: $a_{\epsilon''}^{(\pi^-K^+)} = -0.04 \pm 0.16$ [6]. Even at 90% CL, $a_{\epsilon''}^{(\pi^-K^+)}$ is limited in the range [-0.30,0.22]. Incoporating this result, the allowed regions for the phases γ and δ are further constrained , which is shown in Fig. 3. It is seen that some regions have further been excluded when $a_{\epsilon''}^{(\pi^-K^+)}$ has the value within the 1σ standard deviation. If the values of R_1, R_2 and R_3 are taken to be at 2σ level, $a_{\epsilon''}^{(\pi^-K^+)}$ could be as large as the experimental bound given at 90% CL. From Eq.(23) and (17), the maximum value of $a_{\epsilon''}^{(\pi^+\pi^-)}$ is approximately given by

$$a_{\epsilon''}^{(\pi^+\pi^-)}|_{max} \simeq 0.59\frac{f_\pi^2}{f_K^2} R_1^{(max)} \times a_{\epsilon''}^{(\pi^-K^+)}|_{max}$$

$$\lesssim 0.40, \quad \text{(at } 1\sigma \text{ level)} \tag{24}$$

Where R_1 is taken to be within the 1σ standard deviation. The numerical results for $a_{\epsilon''}^{(\pi^+\pi^-)}$

and $a_{\epsilon''}^{(\pi^- K^+)}$ are plotted in Fig.4 and Fig.5 as functions of the ratios R_1 and R_3. It can be seen that for $R_1 \gtrsim 2.65$ one has $|a_{\epsilon''}^{(\pi^+\pi^-)}| > |a_{\epsilon''}^{(\pi^- K^+)}|$.

In conclusion, assuming $SU(3)$ symmetry for the strong phases and ignoring the relative small EWP effects in the $B \to \pi^-\pi^+, \pi^0\pi^+, \pi^- K^+, \pi^- \bar{K}^0$ decays, a model independent approach is proposed to extract the weak phase γ and the strong final interacting phase δ. From the present data a negative $\cos\gamma \cos\delta$ is favored. Two solutions for the phases γ and δ have been obtained at 1σ level of the current experimental data, though their allowed regions have been strongly restricted, there remain large uncertainties, two interesting allowed regions for the phases γ and δ have been obtained at the 1σ level. The numerical values of the phases γ and δ have been found to lie in the regions: $35° \lesssim \gamma \lesssim 62°$ and $106° \lesssim \delta \lesssim 180°$ or $86° \lesssim \gamma \lesssim 151°$ and $0° \lesssim \delta \lesssim 75°$ We would like to point out that with large uncertainties of the current experimental data at more than 1σ level, one cannot exclude solutions with a small strong phase δ and the values of γ constrained from V_{ub}, $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings and indirect CP-violating parameter ϵ_K within the standard model. The direct CP asymmetries $a_{\epsilon''}^{(\pi^+\pi^-)}$ and $a_{\epsilon''}^{(\pi^- K^+)}$ in $B \to \pi^+\pi^-$ and $\pi^- K^+$ decays have also be estimated. within the errors of the measurement of $a_{\epsilon''}^{(\pi^- K^+)}$, the maximum value of $a_{\epsilon''}^{(\pi^+\pi^-)}$ could be as large as 40%, a larger value may be possible if R_1 is large. The more precise experimental data in the $B \to \pi\pi$ and πK decays will be very plausible for extracting the important weak phase γ and strong phase δ as well as testing how good of the factorization approach. It may also provide us a possible window for new physics with new CP-violating sources [27] which could change all the constraints arising from the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings, radiative rare B decays $(b \to s\ \gamma)$ and observed CP-violating parameters in the kaon decays as well as from the decay amplitudes of hadrons [28]. Finally, we would like to address that our current results have been obtained by assuming the SU(3) relations among the strong phases and ignoring the EWP effects in the considered four decay modes. To precisely extract the phases γ and δ, one needs to improve not only the experimental measurements but also theoretical approaches which is going to be investigated elsewhere.

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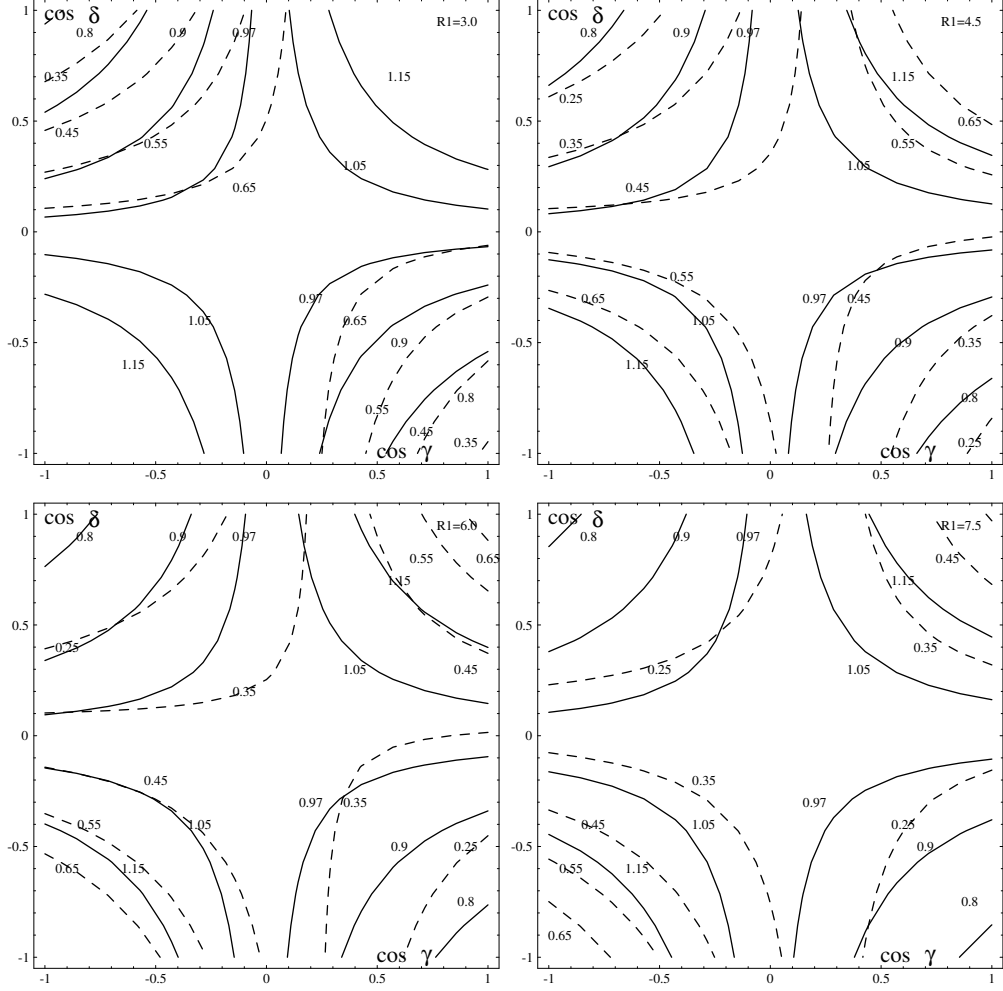
FIG. 1.   The contours of R_2(solid) and R_3(dashed) in $\cos\gamma - \cos\delta$ plane. The five solid(dashed) curves correspond to R_2=0.8, 0.9, 0.97, 1.05, 1.15(R_3= 0.25, 0.35, 0.45, 0.55, 0.65) respectively with R_1 varies from 3.0, 4.5, 6.0 to 7.5.
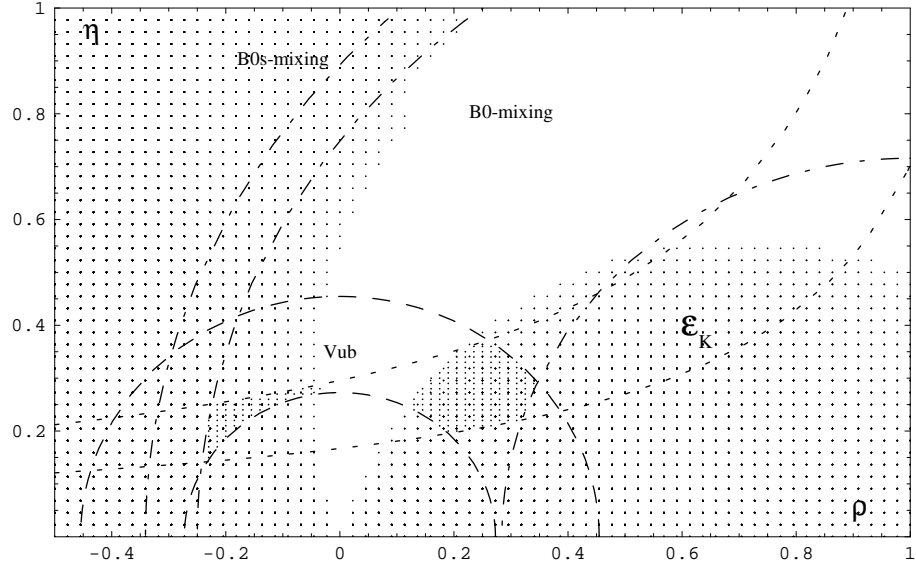
FIG. 2. The allowed region in $\rho - \eta$ plane. The shadowed area corresponds to the allowed region from the constraints of R_1, R_2 and R_3 at 1 σ level. Other constraints are from $B^0_{d,s}, \epsilon_K$ and V_{ub}. The dark area is the allowed region with all the constraints included.
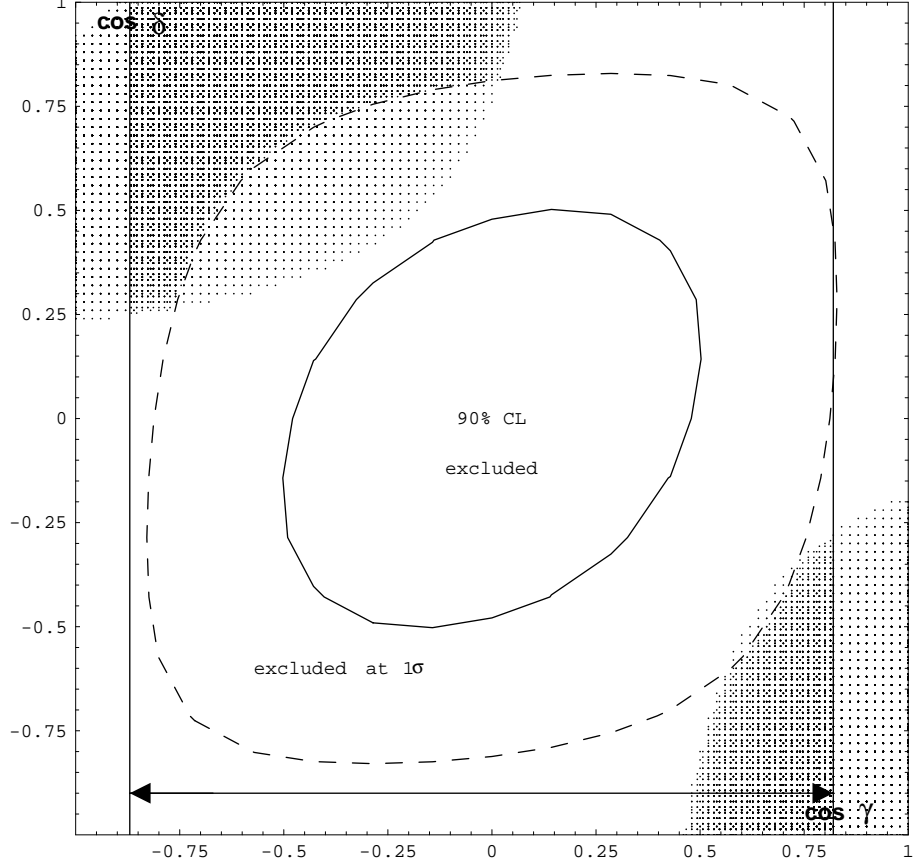
FIG. 3.   The allowed regions of the phases γ and δ in $\cos\delta - \cos\gamma$ plane. The whole shad-owed areas are the allowed region from the constraints of the ratios R_1, R_2 and R_3 at 1σ level with $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$. The regions within the closed curves are corresponding to the ones exluded by the data of $a_{\epsilon''}^{(\pi^- K^+)}$ at 1σ level (dashed one) and at 90% CL (solid one), respectively. The range within the two vertical lines is the allowed range for the phase γ constrained from $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings, ϵ_K and V_{ub}. The dark shadowed regions are the allowed ones for the phases γ and δ from the whole constraints.
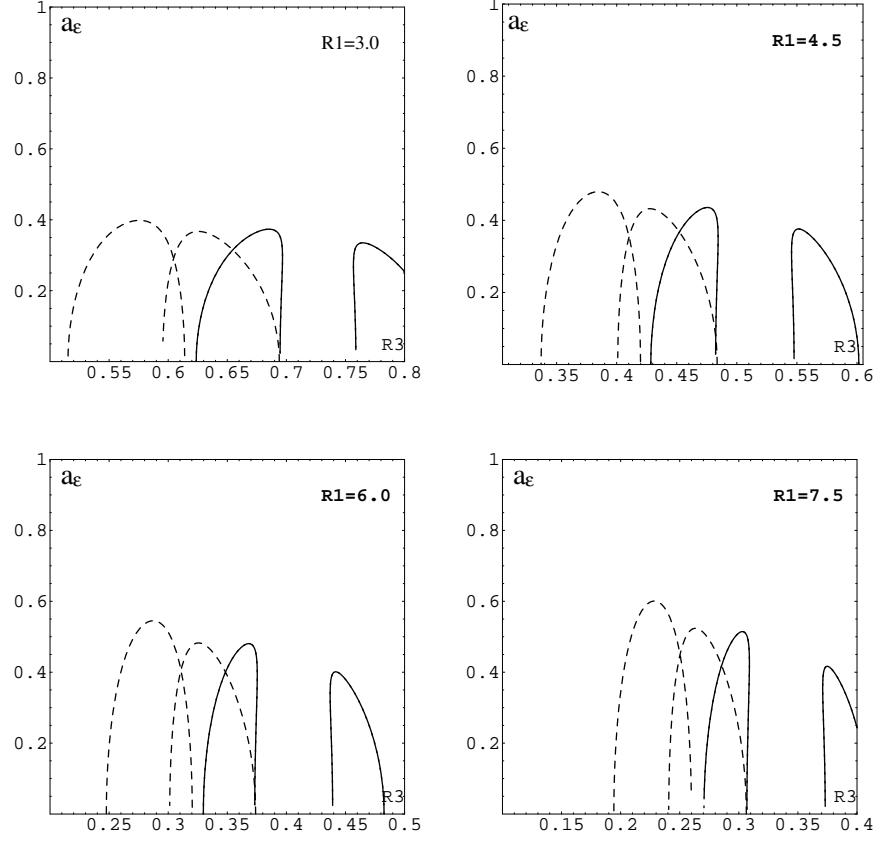
FIG. 4. $a_{\epsilon''}^{(\pi^+\pi^-)}$ vs R_3 with different values of R_1. The dashed and solid curves correspond to R_2=0.95 and 1.05 respectively
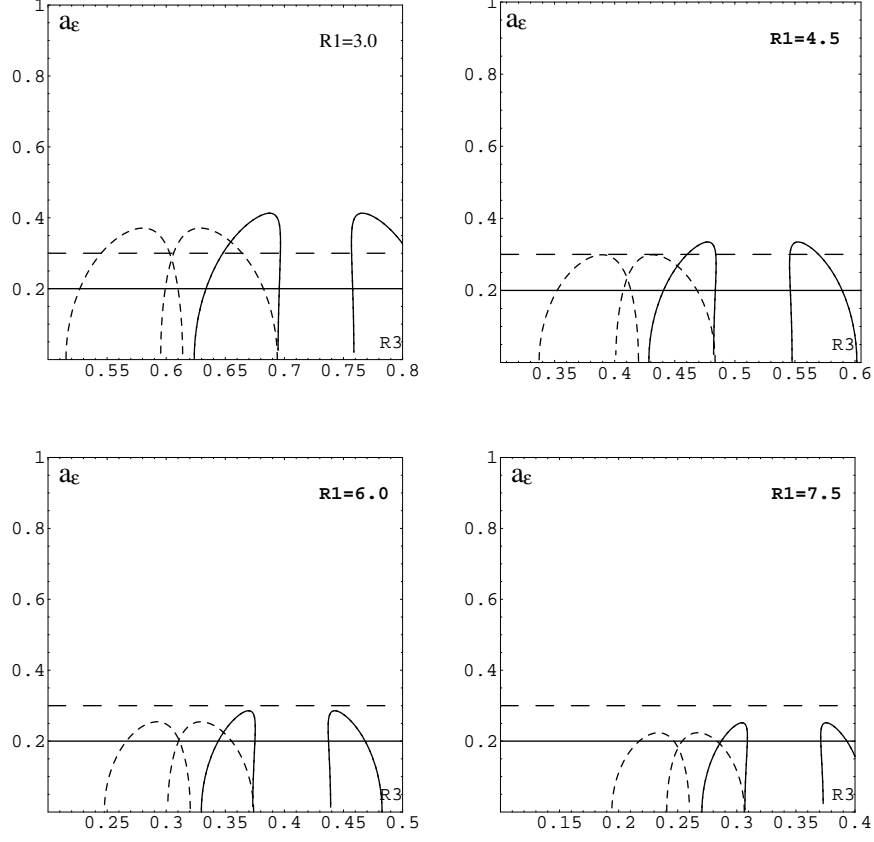
FIG. 5. The same as in Fig.4, but for $|a_{\epsilon''}^{(\pi^- K^+)}|$. The two horizontal lines indicate the upper bound from the CLEO data at 1σ(solid) and 90% CL respectively.